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Energy loss due to small-angle scattering in general relativity

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Abstract. The equations of motion in general relativity using a 'fast motion approach' are worked out explicitly to second order. The derivation from first principles with the assumptions made is given. The application of these equations to small-angle scattering will be given in a later paper.

1. Introduction

The determination of radiation reaction forces is a fundamental problem of any field theory of gravitation. More specifically, the discovery of the binary pulsar PSR 1913+16 and systems such as 4U 1626−67 has led to new interest in the energy loss and the associated damping of binary systems due to gravitational radiation. The recent discovery of gravitational radiation effects (Taylor and Fowler 1979) associated with the period change of the binary pulsar and its apparent agreement with the standard Einstein quadrupole formula has led to even more urgency in attempts to understand radiation reaction forces in general relativity. Since the formula for the energy loss of binary systems has still not been derived consistently using approximation methods within general relativity (Ehlers *et al* 1976, Cooperstock and Hobill 1979, Papapetrou 1974, Cohen 1980), this paper will be the first of a series which will attempt to settle this question in general relativity by the use of approximation methods. In particular, we will present a Poincaré invariant method which avoids the divergence problems associated with 'slow motion' approaches. We will nowhere use a near-zone expansion of the metric in the far zone which is a feature of all the slow motion approaches that have actually been carried out to high enough order to provide terms for radiation reaction forces.

An example of a Poincaré invariant approach is that of Havas and Goldberg (1962). As pointed out in Ehlers *et al* (1976), the equation of motion with radiation reaction terms presented by Havas and Goldberg is consistent but not complete. In fact, two more orders of iteration are needed to obtain complete results for the bound state motions of binary systems. In this paper we present a second-order Lorentz covariant equation of motion for point particles. This is obtained using an extension of the Havas–Goldberg method. A third-order equation of motion will be presented elsewhere.

2. Formalism

I wish to determine a space–time which satisfies Einstein's equation

$$G^{\mu\nu} = 8\pi G T^{\mu\nu} \qquad (c=1) \tag{1}$$

and thus also its consequence

$$\nabla \cdot T = 0. \tag{2}$$

Since in the problems that I will consider the bodies are far apart, I represent them as monopole point particles, so that for one particle

$$T^{\alpha\beta} = m\int_{-\infty}^{\infty} \frac{\dot{Z}^\alpha \dot{Z}^\beta \delta^4 \,\mathrm{d}t}{[g_{\gamma\delta}(-g)\dot{Z}^\gamma \dot{Z}^\delta]^{1/2}} \tag{3}$$

where $\dot{Z} = \mathrm{d}Z^\alpha/\mathrm{d}\tau$, $\mathrm{d}\tau^2 = \eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$, $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ and δ^4 is the Dirac distribution on space–time, a biscalar density. For the problems to be considered, $T^{\mu\nu}$ in (1) is a sum of two terms of the form (3), of course.

I replace Einstein's equation (1) with an equivalent form of the field equation (Landau and Lifshitz 1951, Fock 1959)

$$[\tilde{g}^{\mu\nu}\tilde{g}^{\rho\sigma} - \tilde{g}^{\mu\rho}\tilde{g}^{\nu\sigma}]_{,\rho\sigma} = 16\pi G\theta^{\mu\nu} \tag{4}$$

where $\tilde{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}$ with

$$\theta^{\mu\nu} = (-g)(T^{\mu\nu} + \tau^{\mu\nu}) \tag{5}$$

where $T^{\mu\nu}$ is the matter tensor and $\tau^{\mu\nu}$ is the standard Landau–Lifshitz pseudo-tensor with g the determinant of the metric tensor $g_{\mu\nu}$. In addition equation (4) implies

$$\theta^{\mu\nu}_{,\nu} = 0. \tag{6}$$

We now let

$$\tilde{g}^{\mu\nu} = \eta^{\mu\nu} + \gamma^{\mu\nu} \tag{7}$$

where $\eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. Equation (4) becomes

$$[\eta^{\mu\nu}\gamma^{\rho\sigma}_{,\rho\sigma} + \eta^{\rho\sigma}\gamma^{\mu\nu}_{,\rho\sigma} - \eta^{\mu\rho}\gamma^{\nu\sigma}_{,\rho\sigma} - \eta^{\nu\sigma}\gamma^{\mu\rho}_{\rho\sigma}] = 16\pi G\theta^{\mu\nu} - [\gamma^{\mu\nu}\gamma^{\rho\sigma}\gamma^{\mu\rho}\gamma^{\nu\sigma}]_{,\rho\sigma}. \tag{8}$$

In order to solve Einstein's field equations for arbitrary values of the matter variables, we impose the DeDonder coordinate condition

$$\gamma^{\mu\nu}_{,\nu} = 0 \tag{9}$$

which leads to the related field equation

$$\eta^{\rho\sigma}\gamma^{\mu\nu}_{,\rho\sigma} = H^{\mu\nu} \tag{10}$$

$$H^{\mu\nu} = 16\pi G\theta^{\mu\nu} - [\gamma^{\mu\nu}\gamma^{\rho\sigma}\gamma^{\mu\rho}\gamma^{\nu\sigma}]_{,\rho\sigma}. \tag{11}$$

In using the DeDonder coordinates, we are assuming that an isolated system can be described in this coordinate system. Globally this is still an open question for physically interesting systems.

In order to proceed, we convert (10) into an integral equation using the retarded Green function defined on flat space. The use of the retarded Green function is supposed to represent the physical assumption that the system has been isolated. Past work has indicated (Leipold 1976, Leipold and Walker 1977) that whether or not there is any incoming radiation depends on the early motion of the sources. In addition, all the rigorous treatments assume that non-stationary space–times exist which possess a past null infinity in the sense of Penrose (1964) which may or may not be correct (Walker and Will 1979).

In addition, by using the flat space propagator at all stages of the iteration procedure to be discussed later, convergence of the method is made doubtful (Christodoulou and Schmidt1979) and the question of whether or not the solution satisfies the correct boundary condition is still open (Bird and Dixon 1975, Thorne and Kovacs 1975). Lastly, the problem of for what physically realistic systems (scattering, bound states, etc) we can replace the differential equation by an integral one is still an open question (Walker and Will 1979). That the replacement of the correct Green function by the Minkowski space one is probably not a bad approximation is shown in the work of Christodoulou and Schmidt (1979). These authors show that for a finite time interval and for a source of the gravitational field which has a not too dense matter distribution, then the iteration method discussed later in this paper is asymptotic to the exact solution.

With the above problems in mind we convert (10) into an integral equation using the retarded flat space Green function and obtain

$$\gamma^{\mu\nu} = \frac{1}{4\pi}\int \frac{\delta^4(x^0 - x^{0'} - r)H^{\mu\nu}(x')}{r}\,\mathrm{d}^4x' \tag{12}$$

with $r = [(x_1 - x_1')^2 + (x_2 - x_2')^2 + (x_3 - x_3')^2]^{1/2}$ and δ^4 the four-dimensional Dirac delta function.

I solve (12) approximately as follows. Put $\gamma^{\mu\nu} = 0$ on the right-hand side and obtain the linearised field $\gamma_1^{\mu\nu}$ associated with $T(\eta)$ for arbitrary orbits. $\gamma_1^{\mu\nu}$ is proportional to $\varepsilon = 4G/c^2$. Next insert ${}_1\gamma^{\mu\nu}$ into the right-hand side of (12) and formally expand each term of the resulting expansion to second order in ε as a functional of unspecified orbits. The use of point particles implies that the metric contains divergent terms. I postpone the discussion of regularisation until I have obtained the equation of motion.

To get equations of motion, I insert the approximate metric into (2) and keep terms up to the second order in ε only; the resulting equations are Poincaré invariant analogues of the post-Newtonian equation of motion. I assume the coordinate condition (9) to be satisfied to third order as a consequence of our equation of motion. General conditions under which (8) (or (10)) and (2) imply (9) are under investigation by D Christodoulou (private communication); I hope that they will include small-angle scattering and the bound state problem.

Following the above procedure I obtain the law of motion

$$\begin{aligned} m_a(\mathrm{d}/\mathrm{d}\tau_a)[(\eta_{\mu\rho} + {}_1g_{\mu\rho} + {}_2g_{\mu\rho})\dot{a}^\rho - \tfrac{1}{2}\eta_{\mu\rho}\dot{a}^\rho({}_1g_{\alpha\beta}\dot{a}^\alpha\dot{a}^\beta + {}_2g_{\alpha\beta}\dot{a}^\alpha\dot{a}^\beta - \tfrac{3}{4}({}_1g_{\alpha\beta}\dot{a}^\alpha\dot{a}^\beta)^2) \\ -\tfrac{1}{2}\dot{a}^\rho\,{}_1g_{\mu\rho}\,{}_1g_{\alpha\beta}\dot{a}^\alpha\dot{a}^\beta] \\ = \tfrac{1}{2}m_a\dot{a}^\rho\dot{a}^\sigma\partial_\mu[{}_1g_{\rho\sigma} + {}_2g_{\rho\sigma}] - \tfrac{1}{4}m_a\dot{a}^\rho\dot{a}^\sigma\partial_\mu[{}_1g_{\rho\sigma}]\,{}_1g_{\alpha\beta}\dot{a}^\alpha\dot{a}^\beta \end{aligned} \tag{13}$$

$${}_1g_{\alpha\beta} = -{}_1\gamma_{\alpha'\beta'} + \tfrac{1}{2}\eta_{\alpha\beta}\,{}_1\gamma^{\tau}_{\tau'} \tag{14}$$

$${}_2g_{\alpha\beta} = -{}_2\gamma_{\alpha'\beta'} + \tfrac{1}{2}\eta_{\alpha\beta}[{}_2\gamma^{\tau}_{\tau'} + \tfrac{1}{4}({}_1\gamma^{\tau}_{\tau'})^2 - \tfrac{1}{2}\,{}_1\gamma^{\tau}_{\omega'}\,{}_1\gamma^{\omega}_{\tau'}] + {}_1\gamma_{\alpha'\tau'}\,{}_1\gamma^{\tau}_{\beta'} - \tfrac{1}{2}\,{}_1\gamma_{\alpha'\beta'}\,{}_1\gamma^{\tau}_{\tau'} \tag{15}$$

where the primed indices mean that the indices are lowered with the Minkowski metric $\eta_{\mu\nu}$ and the explicit form of the γ's is given below; a^α are the four positions of particle one or two. Equation (13) is the second-order expansion in the parameter

$\varepsilon = 4G/c^2$ of the geodesic law

$$m_a \frac{\mathrm{d}}{\mathrm{d}\tau_a} \frac{g_{\mu\rho}\dot{a}^\rho}{(g_{\alpha\beta}\dot{a}^\alpha\dot{a}^\beta)^{1/2}} = \frac{m_a}{2} \frac{\dot{a}^\rho\dot{a}^\sigma}{(g_{\alpha\beta}\dot{a}^\alpha\dot{a}^\beta)^{1/2}} \partial_\mu g_{\rho\sigma} \tag{16}$$

where the particle trajectories are parametrised by the Minkowski proper time.

Following the above description, equation (10) with the right-hand side expanded up to order ε^2 is

$$\Box\gamma^{\alpha\beta} = -4\pi\varepsilon \sum m_a \int \mathrm{d}\tau_a\, \delta^4(x-a)\dot{a}^\alpha\dot{a}^\beta(1 + \tfrac{1}{2}\gamma_{\tau'\omega'}\dot{a}^\tau\dot{a}^\omega + \tfrac{1}{4}\gamma^{\tau}_{\tau'}) + \gamma^{\rho\sigma}\gamma^{\alpha\beta}_{,\rho\sigma}. \tag{17}$$

Using equations (13), (14), (15), (19) we have as our final finite equation of motion

$$\begin{aligned}
\ddot{C}_\mu = \tfrac{1}{2}\dot{C}^\rho\dot{C}^\sigma\partial_\mu\Big(& -\sum m_a\varepsilon \int \mathrm{d}\tau_a D(x-a)(\dot{a}_\rho\dot{a}_\sigma - \tfrac{1}{2}\eta_{\rho\sigma}) \\
& -\varepsilon^2 \sum\sum m_a m_b \int \mathrm{d}\tau_a \int \mathrm{d}\tau_b D(b-a)D(x-b)\{(\tfrac{1}{2}(\dot{a}\dot{b})^2 + \tfrac{1}{4})\dot{b}_\rho\dot{b}_\sigma \\
& - \tfrac{1}{2}[\dot{a}_\rho\dot{b}_\sigma\dot{a}\dot{b} + \tfrac{1}{4}\eta_{\rho\sigma}(\tfrac{1}{2} - (\dot{a}\dot{b})^2)]\} \\
& -\varepsilon^2 \int \mathrm{d}\tau_a \int \mathrm{d}\tau_b D(x-a) \sum m_a \sum m_b D(x-b) \\
& \times[\tfrac{1}{2}(\dot{a}_\rho\dot{b}_\sigma\dot{a}\dot{b} - \tfrac{1}{4}\eta_{\rho\sigma}) - \dot{a}_\rho(\dot{a}\dot{b})\dot{b}_\sigma - \tfrac{1}{2}\dot{a}_\rho\dot{a}_\sigma] \\
& -\varepsilon^2 \sum m_a \sum m_b \int \mathrm{d}\tau_a \int \mathrm{d}\tau_b D(x-a)D(a-b) \\
& -\tfrac{1}{2}\{\dot{a}_\rho\dot{b}_\sigma\dot{a}\dot{b} + \tfrac{1}{4}\eta_{\rho\sigma}[\tfrac{1}{2} - (\dot{a}\dot{b})^2]\}\Big) \\
& -\tfrac{1}{4}\dot{C}^\rho\dot{C}^\sigma\,\partial_\mu\Big(-\varepsilon\sum m_a D(x-a)\dot{a}_\rho\dot{a}_\sigma + \tfrac{1}{2}\eta_{\rho\sigma}\varepsilon\sum m_a \int \mathrm{d}\tau_a D(x-a)\Big) \\
& \times\Big(-\varepsilon\sum\int \mathrm{d}\tau_b\, m_b\dot{b}_\alpha\dot{b}_\beta D(x-b) + \tfrac{1}{2}\eta_{\alpha\beta}\varepsilon\sum m_b\int \mathrm{d}\tau_b D(x-b)\Big)\dot{C}^\alpha\dot{C}^\beta \\
& -(\mathrm{d}/\mathrm{d}\tau)\Big[-\varepsilon\sum m_a \int \mathrm{d}\tau_a D(x-a)[\dot{a}_\mu(\dot{a}\dot{c}) - \tfrac{1}{4}\dot{C}_\mu - \tfrac{1}{2}\dot{C}_\mu(\dot{a}\dot{c})^2] \\
& -\varepsilon^2\int \mathrm{d}\tau_a \int \mathrm{d}\tau_b \sum\sum m_a m_b D(b-a)D(x-b)[\tfrac{1}{2}(\dot{a}\dot{b})^2 + \tfrac{1}{4}]\dot{b}_\mu(\dot{b}\dot{c}) \\
& -\tfrac{1}{2}\{\dot{a}_\mu\dot{b}_\rho\dot{a}\dot{b} + \tfrac{1}{4}\eta_{\mu\rho}[\tfrac{1}{2} - (\dot{a}\dot{b})^2]\}\dot{C}^\rho - \tfrac{1}{2}\dot{C}_\mu[\tfrac{1}{2}(\dot{a}\dot{b})^2 + \tfrac{1}{4}](\dot{b}\dot{c})^2 \\
& +\tfrac{1}{4}\dot{C}_\mu\{\dot{a}_\alpha\dot{b}_\beta\dot{a}\dot{b} + \tfrac{1}{4}\eta_{\alpha\beta}[\tfrac{1}{2} - (\dot{a}\dot{b})^2]\}\dot{C}^\alpha\dot{C}^\beta \\
& -\varepsilon^2\sum m_a \sum m_b \int \mathrm{d}\tau_a \int \mathrm{d}\tau_b D(x-a)D(x-b)\{\dot{C}_\mu[\tfrac{1}{4}(\dot{a}\dot{c})(\dot{a}\dot{b})(\dot{b}\dot{c}) \\
& -\tfrac{1}{16} - \tfrac{3}{8}[\tfrac{1}{2} - (\dot{a}\dot{c})^2][\tfrac{1}{2} - (\dot{b}\dot{c})^2] - \tfrac{1}{4}(\dot{a}\dot{c})^2\} + a_\mu[\tfrac{1}{2}(\dot{c}\dot{a}) - (\dot{a}\dot{b})(\dot{b}\dot{c})]\Big\} \\
& -\varepsilon^2\sum m_a \sum m_b \int \mathrm{d}\tau_a \int \mathrm{d}\tau_b D(x-a)D(a-b) \\
& \times\Big(-\tfrac{1}{2}\{\dot{a}_\mu\dot{b}_\rho\dot{a}\dot{b} + \tfrac{1}{4}\eta_{\mu\rho}[\tfrac{1}{2} - (\dot{a}\dot{b})^2]\}\dot{C}^\rho
\end{aligned}$$

$$+\tfrac{1}{4}\eta_{\mu\rho}\dot{C}^{\rho}\{\dot{a}_{\alpha}\dot{b}_{\beta}\dot{a}\dot{b}+\tfrac{1}{4}\eta_{\alpha\beta}[\tfrac{1}{2}-(\dot{a}\dot{b})^2]\}\dot{C}^{\alpha}\dot{C}^{\beta}$$

$$-\tfrac{1}{2}\dot{C}^{\rho}\Big(-\sum\varepsilon m_a\int \mathrm{d}\tau_a D(x-a)\dot{a}_{\mu}\dot{a}_{\rho}+\tfrac{1}{2}\eta_{\mu\rho}\varepsilon\sum m_a\int \mathrm{d}\tau_a D(x-a)\Big)$$

$$\times\Big(-\varepsilon\sum m_b\int \mathrm{d}\tau_b D(x-b)\dot{b}_{\alpha}\dot{b}_{\beta}+\tfrac{1}{2}\eta_{\alpha\beta}\varepsilon\sum m_b\int \mathrm{d}\tau_b D(x-b)\Big)\dot{C}^{\alpha}\dot{C}^{\beta}\Big]$$

$$-\tfrac{11}{3}Gm_c(\dddot{C}_{\mu}+\dot{C}_{\mu}\ddot{C}_{\nu}\ddot{C}^{\nu}) \tag{18}$$

where x^{α} is to be evaluated at particle position C^{α}. The sums are to be taken so that in the retarded Green functions the particle positions do not overlap. The above equations are similar in form to those of Bertotti and Plebanski (1960) except we have no infinite terms and that we have given a derivation.

The finite radiation reaction terms $-\frac{11}{3}Gm$ ( ) from the linearised theory are obtained in the simplest manner by the use of Riesz potentials (Rosenblum *et al* 1982). We define the linearised Riesz potential as

$$^{\alpha}{}_{1}g_{\mu\nu}=-\frac{16\pi Gm}{H(\alpha)}\int_{-\infty}^{\tau_0}(\nu_{\mu}\nu_{\nu}-\tfrac{1}{2}\eta_{\mu\nu})s^{\alpha-4}\,\mathrm{d}\tau \tag{19}$$

where $H(\alpha)=2^{\alpha-1}\pi\Gamma[\frac{1}{2}(\alpha-2)]$, $s^{\rho}=x^{\rho}-z^{\rho}(\tau)$, $s^2=\eta_{\rho\sigma}s^{\rho}s^{\sigma}$. Analytic continuation of equation (19) to $\alpha=2$ produces the classical linearised gravitational potential at events not located on the world line. In addition, finite results are obtained when the Riesz potentials are evaluated on the world line and this gives results equivalent to other more laborious methods. In addition we have

$$^{\alpha}(\partial_{\rho}{}_{1}g_{\mu\nu})=\frac{16\pi Gm(4-\alpha)}{H(\alpha)}\int_{-\infty}^{\tau_0}(\nu_{\mu}\nu_{\nu}-\tfrac{1}{2}\eta_{\mu\nu})s_{\rho}s^{\alpha-6}\,\mathrm{d}\tau. \tag{20}$$

We now expand the integrands in expressions (19) and (20) around the retarded point τ_0, using the Taylor series

$$\begin{aligned}&s^{\mu}=-\nu_0^{\mu}\tau-\tfrac{1}{2}\dot{\nu}_0^{\mu}\tau^2-\tfrac{1}{6}\ddot{\nu}_0^{\mu}\tau^3-\cdots \qquad \nu^{\mu}=\nu_0^{\mu}+\dot{\nu}_0^{\mu}\tau+\tfrac{1}{2}\ddot{\nu}_0^{\mu}\tau^2+\cdots\\ &s=-\tau-\tfrac{1}{24}\ddot{\nu}_0^{\rho}\nu_{0\rho}\tau^3 \qquad s^{\mu}\nu^{\nu}-s^{\nu}\nu^{\mu}=\tfrac{1}{2}(\dot{\nu}_0^{\mu}\nu_0^{\nu}-\dot{\nu}_0^{\nu}\nu_0^{\mu})\tau^2+\tfrac{1}{3}(\ddot{\nu}_0^{\mu}\nu_0^{\nu}-\ddot{\nu}_0^{\nu}\nu_0^{\mu})\tau^3+\cdots.\end{aligned} \tag{21}$$

Because of the form of $H(\alpha)$, we can restrict ourselves to the terms in the integrands proportional to $\tau^{\alpha-3}$. Therefore we have (omitting the subscripts zero)

$$^{\alpha}{}_{1}g_{\mu\nu}\stackrel{*}{=}\frac{8(\alpha-2)Gm}{2^{\alpha-1}[\Gamma(\frac{1}{2}\alpha)]^2}\int_{-\infty}^{\tau_0}(-\tau)^{\alpha-3}\frac{\mathrm{d}\nu_{\mu}\nu_{\nu}}{\mathrm{d}\tau}\,\mathrm{d}\tau \tag{22}$$

$$^{\alpha}(\partial_{\rho}{}_{1}g_{\mu\nu})\stackrel{*}{=}\frac{4(4-\alpha)(\alpha-2)Gm}{2^{\alpha-1}[\Gamma(\frac{1}{2}\alpha)]^2}\int_{-\infty}^{\tau_0}(-\tau)^{\alpha-3}\eta_{\rho\sigma}$$

$$\times\Big[\tfrac{1}{3}(\nu_{\mu}\nu_{\nu}-\tfrac{1}{2}\eta_{\mu\nu})\Big(+\ddot{\nu}^{\sigma}-\frac{6-\alpha}{4}\ddot{\nu}^{\beta}\nu_{\beta}\nu^{\sigma}\Big)+\dot{\nu}^{\sigma}\frac{\mathrm{d}\nu_{\mu}\nu_{\nu}}{\mathrm{d}\tau}+\nu^{\sigma}\frac{\mathrm{d}^2\nu_{\mu}\nu_{\nu}}{\mathrm{d}\tau^2}\Big]\,\mathrm{d}\tau \tag{23}$$

and thus for $\alpha=2$ (omitting the left subscript 2)

$$_{1}g_{\mu\nu}=4Gm(\nu_{0\mu}\dot{\nu}_{0\nu}+\dot{\nu}_{0\mu}\nu_{0\nu}) \tag{24}$$

$$\partial_{\rho}{}_{1}g_{\mu\nu}=4Gm\eta_{\rho\sigma}\Big(\tfrac{1}{3}(\nu_{0\mu}\nu_{0\nu}-\tfrac{1}{2}\eta_{\mu\nu})(\ddot{\nu}_0^{\sigma}-\ddot{\nu}_0^{\beta}\nu_{0\beta}\nu_0^{\sigma})+\dot{\nu}_0^{\sigma}\frac{\mathrm{d}\nu_{0\mu}\nu_{0\nu}}{\mathrm{d}\tau}+\nu_0^{\sigma}\frac{\mathrm{d}^2\nu_{0\mu}\nu_{0\nu}}{\mathrm{d}\tau^2}\Big). \tag{25}$$

Inserting the contributions (24) and (25) into expression (13), we obtain the term $-\frac{11}{3}Gm_c(\dddot{C}_\mu + \dot{C}_\mu \ddot{C}_\nu \ddot{C}^\nu)$ in equation (18).

3. Discussion

In the above we have derived the second-order fast motion equations of motion and hopefully point out the major limitations of the approach. The above equations have been applied to the small-angle scattering of two bodies (Rosenblum 1978). There is the possibility of contributions to the energy loss in small-angle scattering from terms from the third-order equations of motion. These equations have been explicitly calculated. Because new techniques are needed to compute the third-order equations of motion and because of the length of the result, these equations will be published later.

Recently (Clarke and Rosenblum 1982), it has been shown that under suitable conditions equations similar to those presented here for the case of small-angle scattering have solutions that both exist and are unique. Work is under way to extend this result to the equations presented here. When this is finally done, more credence will be lent to the entire approach.

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